國立臺北大學 102 學年度碩士班一般入學考試試題

系（所）組別：國際企業研究所

科　　　　目：統計學

☑可 ☐不可使用計算機

一、第 1, 2 題為計算證明題，第 3, 4 題為填充題

(Note. $P(Z \le 2) = 0.977$, $P(Z \le 2.054) = 0.98$, $P(Z \le 2.326) = 0.99$, where Z is standard normally distributed.)

1. Suppose that the random variables $Y_1, \cdots, Y_n$ satisfy $Y_i = \beta x_i + \epsilon_i$, $i = 1, \cdots, n$, where $x_1, \cdots, x_n$ are fixed constants, and $\epsilon_1, \cdots, \epsilon_n$ are identically and independently distributed normal random variables with mean 0 and variance σ^2 (unknown). It is known that the maximum likelihood estimator (MLE) of β is an unbiased estimator of β.
   (a) Show that $\sum_{i=1}^n Y_i / \sum_{i=1}^n x_i$ is also an unbiased estimator of β. (5%)
   (b) Calculate the exact variance of $\sum_{i=1}^n Y_i / \sum_{i=1}^n x_i$ and compare it to the variance of the MLE of β. (10%)

2. Assume that X and Y are independent and standard normally distributed. Find the conditional mean of X, given that $X > Y$.(10%)

3. Assume that in a certain population of married couples the height X of the husband and the height Y of the wife have a bivariate normal distribution. The correlation coefficient between the heights of husbands and wives is 0.6 and the mean male height is 173 cm with standard deviation 6 cm, and the mean female height is 161 cm with standard deviation 5 cm.
   (a) Given that the height of the husband is 188 cm, the probability that his wife has a height between 160.5 and 176.5 cm is ___(1)___. (8%)
   (b) The exact distribution of the average height of a couple is ___(2)___. (7%)

4. Let us say the life of a tire in miles, say X, is normally distributed with mean θ and standard deviation 5000. Past experience indicates that $\theta = 30000$. The manufacturer claims that the tires made by a new process have mean $\theta > 30000$. It is possible that $\theta = 35000$. Check his claim by testing $H_0 : \theta = 30000$ against $H_1 : \theta > 30000$. We observe n independent values of X, say $x_1, \cdots, x_n$, and we reject H_0 if and only if $(1/n)\sum_{i=1}^n x_i > c$. Determine $n =$ ___(3)___ (5%) and $c =$ ___(4)___ (5%) so that the power function $\gamma(\theta)$ of the test has the values $\gamma(30000) = 0.01$ and $\gamma(35000) = 0.98$.

二、填充題

1. How would you handle an influential observation in simple linear regression analysis? ___5%___

2. Define ΔP as the daily change for the value of a portfolio in one day. Suppose that one-day volatility of ΔP is 2 and ΔP from one day to the next day are independent. The estimate of 4-day volatility for ΔP is ___5%___ Suppose that daily changes ΔP have first-order correlation with 0.12. The 4-day volatility for ΔP will be ___10%___.

3. A simple regression analysis was performed on a random sample, generating the following summary statistics SSx =250, SSy =700, SSxy =400, and n=14. The F-value for testing a significant relationship for simple regression analysis is ___10%___

4. An automobile dealer conducted a test to determine if the time in minutes needed to complete a minor engine tune-up depends on the five types of analyzers is used. Because tune-up time varies among three types of cars which were used as blocks in the experiment. The data were obtained: SST=430, SSTR=310, SSBL=85. The F-test value for testing any significant differences in the mean tune-up time among five analyzers is ___10%___ If the block effect is ignored, the F-test value for testing any significant differences in the mean tune-up time among five analyzers for one-way ANOVA is ___10%___

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